

A red double-decker bus is the central focus of the image, positioned on a city street. The bus is a classic model with a large front grille and multiple windows. Its destination sign displays the number '38' and the text 'COPTON FORD'. The background is heavily blurred, showing indistinct shapes of buildings and other vehicles, which creates a sense of motion and depth. A large, semi-transparent red rectangle is overlaid on the left side of the image, containing the chapter title. Another red rectangle is at the bottom, containing the chapter's theme.

# **Chapter 2**

**SUFFICIENCY & COMPLETENESS**



# SUFFICIENCY

Idea → the reduction of a data set to more concise set of statistics with no loss of information about the unknown parameter

→  $S$  will be considered a “sufficient” statistics for  $\theta$  if  $P(T|S)$  does not involve  $\theta$

Example 10.1.1

A coin is tossed  $n$  times, find  $S$  that sufficient for  $\theta$  !



## Definition 10.2.1

Let  $X = (X_1, X_2, \dots, X_n)$  have joint pdf  $f(x, \theta)$  and let  $S = (S_1, S_2, \dots, S_n)$  be a  $k$ -dimensional statistics. Then  $S_1, S_2, \dots, S_n$  is a set of jointly sufficient statistics for  $\theta$  if for any other vector of statistics,  $T$ , the conditional pdf of  $T$  given  $S = s$ , denoted by  $f_{T|s}(t)$  does not depend on  $\theta$ . In the one-dimensional case, say that  $S$  is a sufficient statistic for  $\theta$

### Def. 10.2.2

A set of statistics is called a minimal sufficient set if the member of the set are jointly sufficient for the parameters and if they are function of every other set of jointly sufficient statistics



## Example 10.2.1

Consider  $X_i \sim \text{EXP}(\theta)$

Then find  $S$  that sufficient for  $\theta$



## Th. 10.2.1 Factorization Criterion

If  $X_1, X_2, \dots, X_n$  have joint pdf  $f(x_1, x_2, \dots, x_n)$  and if  $S = (S_1, S_2, \dots, S_k)$  are jointly sufficient for  $\theta$  i.o.i

$$f(x_1, x_2, \dots, x_n; \theta) = g(s; \theta)h(x_1, x_2, \dots, x_n)$$

where  $g(s; \theta)$  doesn't depend on  $x_1, x_2, \dots, x_n$  except through  $s$  and  $h(x_1, x_2, \dots, x_n)$  doesn't involve  $\theta$



## Example

1. if  $X_i \sim \text{BIN}(1, \theta)$  then find  $s$
2. if  $X_i \sim \text{UNIF}(0, \theta)$  then find  $s$
3. if  $X_i \sim N(\mu, \sigma^2)$  then find  $s$

Do the exercise at Bain, page 352, number 1, 2 and 3 !