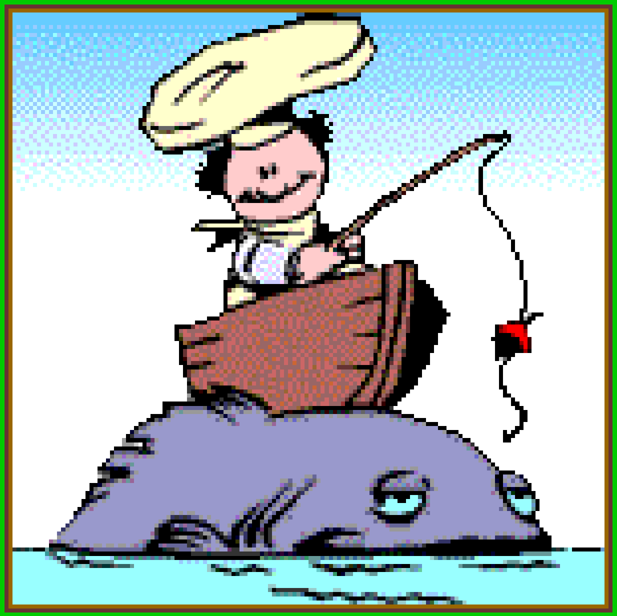
# Why this can be happen to me?

Can you think, who'll be the faster catch the fish??

# Chapter 3

## Special Distribution

What’s distribution we learn for????



# An Applied Example about Distribution ; discrete/ continue

Premium value in insurance industry

# DISCRETE UNIFORM DISTRIBUTION

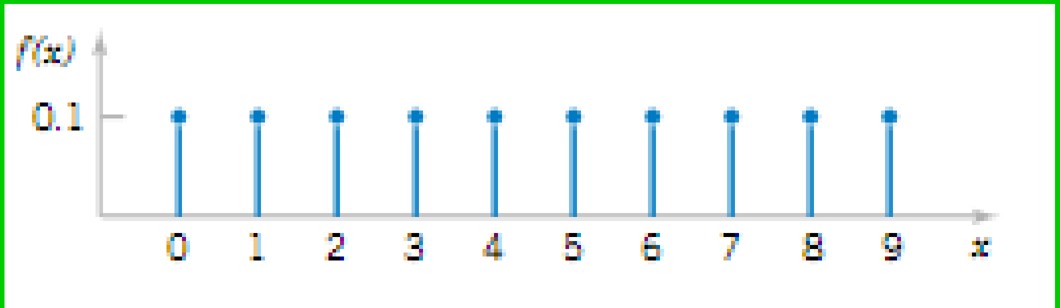
A random variable $X$ has a **discrete uniform distribution** if each of the $n$ values in its range, say, $x_1, x_2, \ldots, x_n$, has equal probability. Then,

$$f(x_i) = 1/n \qquad (3\text{-}5)$$

Example :

The first digit of a part's serial number is equally likely to be any one of the digits 0 through 9. If one part is selected from a large batch and X is the first digit of the serial number, X has a discrete uniform distribution with probability 0.1 for each value.

R={0,1,....9} → f(x)=0.1 for each value in R



# Mean & Variance discrete UNIFORM

Suppose $X$ is a discrete uniform random variable on the consecutive integers $a, a+1, a+2, \ldots, b$, for $a \le b$. The mean of $X$ is

$$\mu = E(X) = \frac{b+a}{2}$$

The variance of $X$ is

$$\sigma^2 = \frac{(b-a+1)^2 - 1}{12} \qquad (3\text{-}6)$$

# Example

Let the random variable Y denote the proportion of the 48 voice lines that are in use at a particular time. Assume that X is a discrete uniform random variable with a range of 0 to 48.

then

E(X)=(48+0)/2=24

$$\sigma = \left\{ \left[ (48 - 0 + 1)^2 - 1 \right] / 12 \right\}^{1/2} = 14.14$$



Can u guess me!
Male or female??

# Bernoulli & Binomial Distribution

- A trial with only two possible outcome → Bernoulli Trial
- Assumed that the trial that constitute the random experiment are independent
- This implies that the outcome from one trial has no effect on the outcome to be obtained from any other trial
- It often reasonable to assume that the probability of a success in each trial is constant

A rV X, if an experiment can result only in “success” (E) or failure (E’) then the corresponding Bernoulli rV is :

$$X(e) = \begin{cases} 1, \text{if } e \in E \\ 0, \text{if } e \notin E' \end{cases}$$

The pdf of X is given by f(0)=q, f(1)=p.

Pdf of Bernoulli distribution can be expressed as :

$$f(x) = p^x q^{1-x}, x = 0{,}1$$

Ex.Rolls of a four sided die. A bet is placed that a 1 will occur on a single roll of the die

# Binomial Distr

A random experiment consists of $n$ Bernoulli trials such that

(1) The trials are independent

(2) Each trial results in only two possible outcomes, labeled as "success" and "failure"

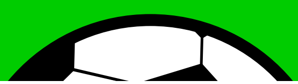
(3) The probability of a success in each trial, denoted as $p$, remains constant

The random variable $X$ that equals the number of trials that result in a success has a **binomial random variable** with parameters $0 < p < 1$ and $n = 1, 2, \ldots$. The probability mass function of $X$ is

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, \ldots, n \tag{3-7}$$

constants $a$ and $b$, the binomial expansion is

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$



If $X$ is a binomial random variable with parameters $p$ and $n$,

$$\mu = E(X) = np \qquad \text{and} \qquad \sigma^2 = V(X) = np(1-p)$$

# Example

Each sample of water has a 10% chance of containing a particular organic pollutant. Assume the samples are independent with regard to the presence of the pollutant.
Find the probability that in the next 18 samples, exactly 2 contain the pollutant.

Let X=the number of samples that contain the pollutant in the next 18 samples analyzed.
Then X is a binomial rV with p=0.1 and n=18
Therefore :

$$P(X=2)=\binom{18}{2}(0.1)^2(0.9)^{16}$$

# Geometric Distribution

- Is a distribution arising from Bernoulli trials is the number of trials to the first occurrence of success
- Ex:

The probability that a bit transmitted through a digital transmission channel is received in error is 0.1. Assume the transmissions are independent event and let the rV X denote the number of bits transmitted until the first error

P(X=5) is the probability that the first four bits are transmitted correctly and the fifth bits is in error

→ Denoted : {OOOOE} where O denotes an okay bit

→ Because the trial are independent and the probability of a correct transmission is 0.9 then

$$P(X=5)=P(OOOOE)=0.9^4 0.1=0.066$$

# Definition

In a series of Bernoulli trials (independent trials with constant probability $p$ of a success), let the random variable $X$ denote the number of trials until the first success. Then $X$ is a **geometric random variable** with parameter $0 < p < 1$ and

$$f(x) = (1-p)^{x-1}p \qquad x = 1, 2, \ldots \tag{3-9}$$

If $X$ is a geometric random variable with parameter $p$,

$$\mu = E(X) = 1/p \qquad \text{and} \qquad \sigma^2 = V(X) = (1-p)/p^2 \tag{3-10}$$

# Definition NEGATIVE BINOMIAL

In a series of Bernoulli trials (independent trials with constant probability $p$ of a success), let the random variable $X$ denote the number of trials until $r$ successes occur. Then $X$ is a **negative binomial random variable** with parameters $0 < p < 1$ and $r = 1, 2\ 3, \ldots,$ and

$$f(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r \qquad x = r, r+1, r+2, \ldots. \qquad \text{(3-11)}$$

If $X$ is a negative binomial random variable with parameters $p$ and $r$,

$$\mu = E(X) = r/p \qquad \text{and} \qquad \sigma^2 = V(X) = r(1-p)/p^2 \qquad \text{(3-12)}$$

# Hypergeometric Distribution

A set of $N$ objects contains

$K$ objects classified as successes

$N - K$ objects classified as failures

A sample of size $n$ objects is selected randomly (without replacement) from the $N$ objects, where $K \leq N$ and $n \leq N$.

Let the random variable $X$ denote the number of successes in the sample. Then $X$ is a **hypergeometric random variable** and

$$f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \qquad x = \max\{0, n + K - N\} \text{ to } \min\{K, n\} \qquad (3\text{-}13)$$

If $X$ is a hypergeometric random variable with parameters $N$, $K$, and $n$, then

$$\mu = E(X) = np \qquad \text{and} \qquad \sigma^2 = V(X) = np(1-p)\left(\frac{N-n}{N-1}\right) \qquad (3\text{-}14)$$

where $p = K/N$.

## Example

A batch of parts contains 100 parts from a local supplier of tubing and 200 parts from a supplier of tubing in the next state. If four parts are selected randomly and without replacement, what is the probability they are all from the local supplier?

Let $X$ equal the number of parts in the sample from the local supplier. Then, $X$ has a hypergeometric distribution and the requested probability is $P(X = 4)$. Consequently,

$$P(X = 4) = \frac{\binom{100}{4}\binom{200}{0}}{\binom{300}{4}} = 0.0119$$

What is the probability that two or more parts in the sample are from the local supplier?

$$P(X \geq 2) = \frac{\binom{100}{2}\binom{200}{2}}{\binom{300}{4}} + \frac{\binom{100}{3}\binom{200}{1}}{\binom{300}{4}} + \frac{\binom{100}{4}\binom{200}{0}}{\binom{300}{4}}$$
$$= 0.298 + 0.098 + 0.0119 = 0.408$$

What is the probability that at least one part in the sample is from the local supplier?

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{\binom{100}{0}\binom{200}{4}}{\binom{300}{4}} = 0.804$$

# definition

Given an interval of real numbers, assume counts occur at random throughout the interval. If the interval can be partitioned into subintervals of small enough length such that

(1) the probability of more than one count in a subinterval is zero,

(2) the probability of one count in a subinterval is the same for all subintervals and proportional to the length of the subinterval, and

(3) the count in each subinterval is independent of other subintervals, the random experiment is called a **Poisson process.**

The random variable $X$ that equals the number of counts in the interval is a **Poisson random variable** with parameter $0 < \lambda$, and the probability mass function of $X$ is

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, 2, \ldots \tag{3-15}$$

If $X$ is a Poisson random variable with parameter $\lambda$, then

$$\mu = E(X) = \lambda \quad \text{and} \quad \sigma^2 = V(X) = \lambda \tag{3-16}$$